Embedding Based Models – Lecture 17.2

- Embedding-based models build embeddings for entities and relations to predict tuples.
- Simplest case: relations between two entities.
  E.g., predicting the rating for a person on a movie (collaborative filtering).
- Knowledge graphs: predict subject-verb-object triples.

Learning a relation between two entities

A relation between users and items (movies). From Movielens:

User	Item	Rating	Timestamp
196	242	3	881250949
186	302	3	891717742
22	377	1	878887116
244	51	2	880606923
...	...	...	

- Netflix: 500K users, 17k movies, 100M ratings (withdrawn).
- Movielens: multiple datasets from 100K to 25M ratings, with links to IMDB, plus some user properties

$\widehat{r_{ui}}$ = predicted rating of user u on item i

Es = set of (u, i, r) tuples in the training set (ignoring timestamp)

Minimize sum squares error:

$$\sum_{(u,i,r)\in Es} (\widehat{r_{ui}} - r)^2$$

Learning Relational Models with Latent Variables

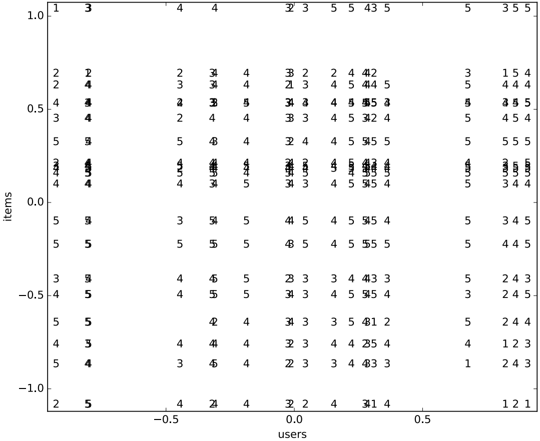
- Predict same for all ratings: $\widehat{r_{ui}} = \mu$
- Adjust for each user and item: $\widehat{r_{ui}} = \mu + b_1[u] + b_2[i]$
  Question: $b_1[u]$ could be negative even though all ratings of u are higher that average. How?
- Question: How can this be used to personalize recommendations?
- One latent feature: f_i for each item and g_u for each user

$$\widehat{r_{ui}} = \mu + b_1[u] + b_2[i] + f_1[u] * f_2[i]$$

  - Positive $f_1[u]$ forms a soft clustering of users.
  - Positive $f_2[i]$ forms a soft clustering of items.
  - How do these clusterings interact?
  - What about negative values?

What is being learned? (Single latent feature)

for each (u, i, r): r is plotted at point $(f_1[u], f_2[i])$.



`aipython: relnCollFilt.py`

What pattern would you expect?

Learning Relational Models with Multiple Latent Properties

- k latent features (Python notation):

$$\widehat{r_{ui}} = \mu + b_1[u] + b_2[i] + \sum_f E_1[u][f] * E_2[i][f]$$

  $E_1[u]$ is the user embedding, a vector of numbers.
  $E_2[i]$ is the item embedding, a vector of numbers.

- Regularize parameters except μ.

Regularizing

- $L2$ regularization, minimize:

$$\left(\sum_{(u,i,r)\in Es} (\widehat{r_{ui}} - r)^2\right) + \lambda \sum_{parameter\ p} p^2$$

$L2$ regularization

Minimize:

$$\left(\sum_{(u,i,r)\in Es}(\mu + b_1[u] + b_2[i] + \sum_k E_1[u][f] * E_2[i][f] - r)^2\right)$$
$$+ \lambda\left(\sum_i(b_1[u]^2 + \sum_f E_1[u][f]^2) + \sum_u(b_2[i]^2 + \sum_f E_2[i][f]^2)\right)$$

where λ is a regularization parameter.

To find minimizing parameters:

- Gradient descent
- Iterative least squares: fix one of E_1 and E_2; the problem is ridge regression in the other.

$\mu :=$ average rating
assign $E_1[u][f]$, $E_2[i][f]$ randomly and assign $b_1[i], b_2[u]$ arbitrarily
repeat:
  for each $(u, i, r) \in Es$:
    $e := \mu + b_1[i] + b_2[u] + \sum_k E_1[u][f] * E_2[i][f] - r$
    $b_1[i] := b_1[i] - \eta * e$
    $b_2[u] := b_2[u] - \eta * e$
    for each feature f:
      $E_1[u][f] := E_1[u][f] - \eta * e * E_2[i][f]$
      $E_2[i][f] := E_2[i][f] - \eta * e * E_1[u][f]$
  for each item i:
    $b_1[i] := b_1[i] - \eta * \lambda * b_1[i]$
    for each feature f:
      $E_1[u][f] := E_1[u][f] - \eta * \lambda * E_1[u][f]$
  for each user u:
    $b_2[u] := b_2[u] - \eta * \lambda * b_2[u]$
    for each feature k:
      $E_2[i][f] := E_2[i][f] - \eta * \lambda * E_2[i][f]$

Variations

- What is you want to predict Boolean *rating* > 3?
  - Use sigmoid.
  - What should we minimize?
  - How does the algorithm change?
- What if we want to predict *rated*, where $rated(u, i)$ is true if $(u, i, r) \in Es$ for some r?
  - There are no negative examples!
  - Use k random examples for each positve example!
    but the average probability is $1/k$, which is not derived from the data.

Knowledge Graphs and Triples

- A knowledge graph is defined in term of triples of the form (s, r, o), with subject s, relation (verb) r, and object o
- The extension of matrix factorization to triples is polyadic decomposition:

$$\widehat{p}((s,r,o)) = sigmoid(\mu + b_1[s] + b_2[r] + b_3[o]$$
$$\sum_f E_1[s][f] * E_2[r][f] * E_3[o][f])$$

  - a global bias μ
  - two biases for each entity e: $b_1[e]$ used when e is in the first position and $b_3[e]$, for when e in third position
  - a bias for each relation r, namely $b_2[r]$
  - matrixes E_1 and E_3
    $E_1[e]$ is subject embedding for entity $e \longrightarrow$ latent properties
    $E_3[e]$, is object embedding entity e
  - matrix E_2, where $E_2[r]$ is relation embedding for relation r.

All embeddings $E_1[e]$, $E_2[r]$ and $E_3[e]$ are the same length.

Learning Polyadic Decomposition

- To optimize log loss with $L2$ regularization:
  same as previous algorithm with a different predictor and more parameters to tune and regularize.
- Requires negative examples, but knowledge graphs don't have negative examples.
- Regularize μ or provide made-up negative examples.
- But, not all relations have same number of negative examples, eg. "*married-to*" vs "*has-streamed*".

Improving Polyadic Decomposition

- Suppose triples are of the form $(u, likes, m)$ and $(m, directed\_by, d)$.
  How can we represent "Sam likes movies directed by Bong Joon-ho"?
  The subject and object embeddings for movies are independent of each other, so this cannot be represented or learned.
- Solution: also represent $(m, likes^{-1}, u)$ and $(d, directed\_by^{-1}, m)$.
- polyadic decomposition with inverses:

$$\widehat{p}(h, r, t) = \frac{1}{2}(\widehat{pd}(h, r, t) + \widehat{pd}(t, r^{-1}, h))$$

where $\widehat{pd}$ is the prediction from the polyadic decomposition.

What does polyadic decomposition learn?

- The polyadic decomposition is fully expressive: it can represent (with error less than any ϵ), any relation.
- Initially assume subject and object embeddings are non-negative and bounded.
- Each embedding position in the subject/object embedding forms a soft clustering of entities.
- For each embedding position, a relation with a high value ($\gg 0$) in that position, the entities in the subject soft cluster are related to the entities in the object soft cluster. (The product is only high when all three are high).
- A relation with a value $\ll 0$ in a position forms exceptions.
- For possibly negative subject and object embeddings:
  product of even number of negative values is positive
  product of odd number of negative values is negative
- The addition lets these values be combined.

What does polyadic decomposition learn?

- The polyadic decomposition makes predictions by clustering entities in different ways.
- It learns about each entity; embeddings are used to predict interactions.
- It does not learn general knowledge that can be applied to other populations.