Lecture 17.4: Relational Representations

- Probabilistic Logic Programs
- Weighted Logical Formulae
- Graph Neural Networks
- Existence and Identity Uncertainty

Probabilistic Logic Programs

- the model is described in terms of a logic program with parametrized independent noise variables.
- Plates correspond to logical variables.
- Parametrized random variables are represented as logical atoms,
- A Turing-complete language for relational probabilistic models.
- Extends Datalog / logic programs to include probabilities.

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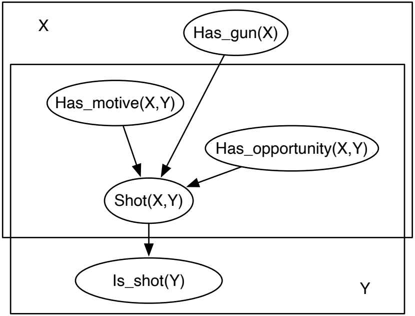

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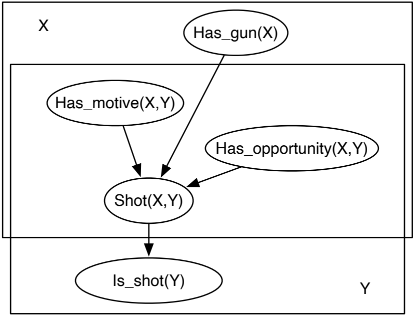

$$is\_shot(Y) \leftarrow shot\_by\_no\_one(Y)$$

$$is\_shot(Y) \leftarrow shot(X, Y) \wedge shot\_succeeds(X, Y)$$

Each ground instance of $shot\_by\_no\_one(Y)$ and $shot\_succeeds(X, Y)$ are independent noise variables.

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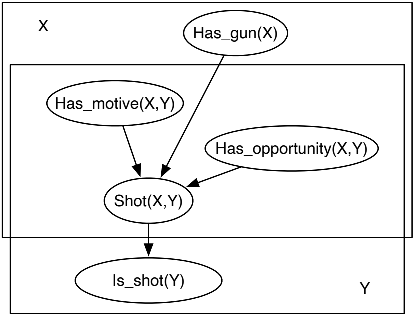

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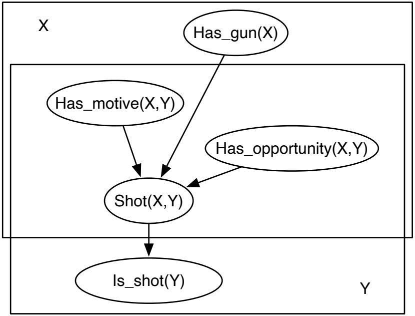

$$shot(X,Y) \leftarrow has\_motive(X,Y) \wedge has\_gun(X)$$
$$\wedge\, has\_opportunity(X,Y) \wedge actually\_shot(X,Y).$$

$P(actually\_shot(X,Y))$ is the probability that X would shoot Y if they had a motive, gun, and opportunity.

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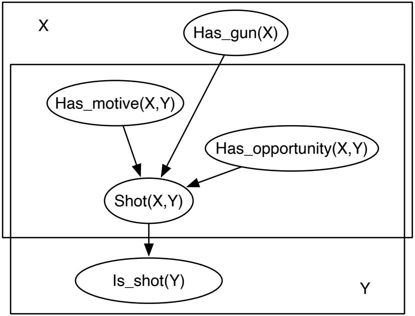

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- Other rules could cover other cases, such as where X doesn't have a motive.

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- In Markov logic networks (MLNs), the measure of a world is proportional to the exponential of the sum of the weights of the formulae true in the world.
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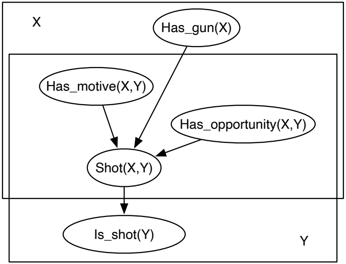
- In relational logistic regression, the weighted formulae are used to define conditional probabilities.

Weighted First-Order Logic Formulas



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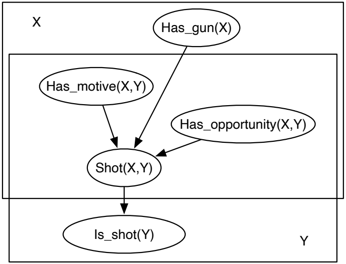

MLNs provide an undirected model, e.g.,

$$(is\_shot(Y), w_0)$$

$$(is\_shot(Y) \vee \neg shot(X, Y), w_1)$$

$$(shot(X, Y) \vee \neg has\_motive(X, Y) \vee \neg has\_gun(X)$$
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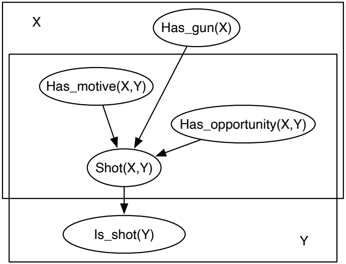

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$P(is\_shot(v) \mid shot(p_1, v), \ldots shot(p_n, v))$ is logistic regression if $p_1, \ldots, p_n$ are all the individuals.

Graph Neural Networks

- Graph neural networks are neural networks that act on graph data.
- Each node has an embedding that is inferred from parametrized linear functions and activation functions of the node's neighbors, and their neighbors, to some depth.
- A relational graph convolutional network (R-GCN) is used to learn embeddings for knowledge graphs, where nodes are entities and arcs are labelled with relations.

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- Called *convolutional* because the same learnable parameters are used for each entity.

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- $W_r^{(L)}$ is a matrix for relation r for layer L, which is multiplied by the vector $h_n^{(L)}$ for each neighbor n.
- $C_{e,r}$ is a normalization constant, such as $|\{n : (e,r,n) \in KG\}|$, which gives an average for each relation.

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  - $W_r^{(L)}$, can be represented as a linear combination of a few learned basis matrices which are shared among the relations
  - the weights in the linear combination depend on the relation.
- Question: Is summing or avaraging an appropriate way to aggegate the embeddings of related entities? Would something else be more appropriate?

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- Number uncertainty concerns uncertainty as how many entities fit a description

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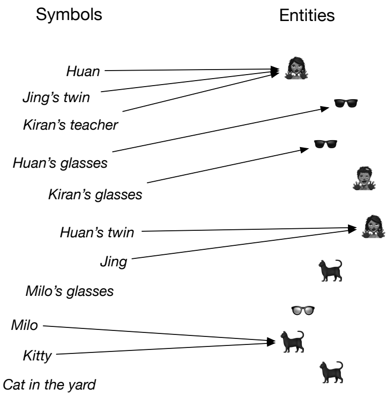

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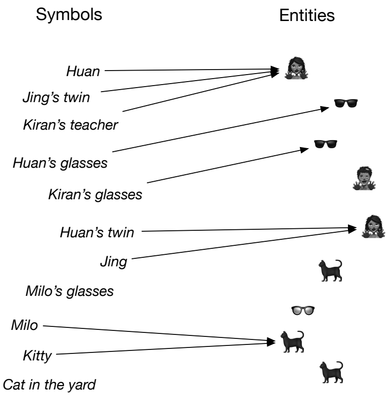

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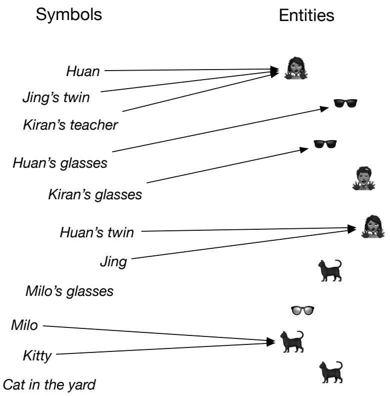

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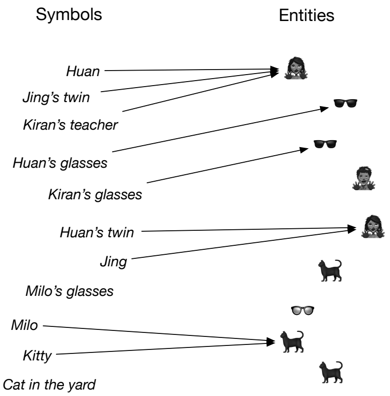

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- *Milo's glasses* do not exist; Milo doesn't have a pair of glasses.

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- Use Markov-chain Monte Carlo (MCMC): given a partition, entities can be moved to different partitions or to new partitions.

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